TRIGONOMETRIC IDENTITIES

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\qquad$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

$\frac{\sin\theta}{\cos\theta} = \tan\theta$ $\qquad$ $\frac{\cos\theta}{\sin\theta} = \cot\theta$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$\sin\theta = \cos(90° - \theta)$ $\qquad$ $\cos\theta = \sin(90° - \theta)$

$\tan\theta = \cot(90° - \theta)$ $\qquad$ $\cot\theta = \tan(90° - \theta)$

$\sec\theta = \csc(90° - \theta)$ $\qquad$ $\csc\theta = \sec(90° - \theta)$

Sum and Difference Identities

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Opposite-Angle Identities

$\sin(-A) = -\sin A$ $\qquad$ $\cos(-A) = \cos A$

Double-Angle Identities

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Half-Angle Identities

$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$

$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$

$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$, $\cos a \neq -1$

Symmetry Identities

The following trigonometric identities hold for any integer k and all values of A.

Case 1: $\sin(A + 360k°) = \sin A$
$\cos(A + 360k°) = \cos A$

Case 2: $\sin(A + 180°(2k - 1)) = -\sin A$
$\cos(A + 180°(2k - 1)) = -\cos A$

Case 3: $\sin(360k° - A) = -\sin A$
$\cos(360k° - A) = \cos A$

Case 4: $\sin(180°(2k - 1) - A) = \sin A$
$\cos(180°(2k - 1) - A) = -\cos A$

FORMULAS

CHAPTER 1 (pp. 4–65)

Slope $m = \frac{y_1 - y_2}{x_1 - x_2}$

CHAPTER 2 (pp. 66–125)

Determinant

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Inverse of a Second Order Matrix

$$A^{-1} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}\begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix}$$

CHAPTER 3 (pp. 126–203)

Direct Variation $y = kx^n, n > 0$

Inverse Variation

$x^ny = k$ or $y = \frac{k}{x^n}, n > 0$

Joint Variation $y = kx^nz^n$, where $x \neq 0, z \neq 0$, and $n > 0$

CHAPTER 4 (pp. 204–273)

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

CHAPTER 5 (pp. 276–341)

Trigonometric Ratios

$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}}$

Inverse Trigonometric Ratios

$\csc\theta = \frac{1}{\sin\theta}$ or $\frac{\text{hypotenuse}}{\text{side adjacent}}$

$\sec\theta = \frac{1}{\cos\theta}$ or $\frac{\text{hypotenuse}}{\text{side adjacent}}$

$\cot\theta = \frac{1}{\tan\theta}$ or $\frac{\text{side adjacent}}{\text{side opposite}}$

Trigonometric Functions of an Angle in Standard Position

$\sin\theta = \frac{y}{r}$ $\quad\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad\cot\theta = \frac{x}{y}$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a Triangle

$K = \frac{1}{2}bc\sin A$

$K = \frac{1}{2}c^2\frac{\sin A\sin B}{\sin C}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Hero's Formula

$K = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a + b + c)$

CHAPTER 6 (pp. 342–419)

Degree/Radian Conversion

1 radian $= \frac{180}{\pi}$ degrees

1 degree $= \frac{\pi}{180}$ radians

Length of an Arc $s = r\theta$

Area of a Circular Sector

$A = \frac{1}{2}r^2\theta$

Angular Velocity $\omega = \frac{\theta}{t}$

Linear Velocity $v = r\frac{\theta}{t}$

CHAPTER 7 (pp. 420–483)

Distance from a Point to a Line

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

CHAPTER 8 (pp. 484–549)

Magnitude of $\overrightarrow{P_1P_2}$ in Two Dimensions

$|\overrightarrow{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Magnitude of $\overrightarrow{P_1P_2}$ in Three Dimensions

$|\overrightarrow{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Inner Product of Vectors in a Plane

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_1b_1 + a_2b_2$

Inner Product of Vectors in Space

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_1b_1 + a_2b_2 + a_3b_3$

Cross Product of Vectors in Space

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\vec{\mathbf{k}}$$

Parametric Equations for the Path of a Projectile

$x = t|\vec{\mathbf{v}}|\cos\theta$ and

$y = t|\vec{\mathbf{v}}|\sin\theta - \frac{1}{2}gt^2$

CHAPTER 9 (pp. 552–613)

Distance Formula in Polar Plane

$P_1P_2 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}$

Conversion from Polar Coordinates to Rectangular Coordinates

$x = r\cos\theta, y = r\sin\theta$

Conversion from Rectangular Coordinates to Polar Coordinates

$r = \sqrt{x^2 + y^2}$

$\theta = \text{Arctan}\,\frac{y}{x}$, when $x > 0$

$\theta = \text{Arctan}\,\frac{y}{x} + \pi$, when $x < 0$

Absolute Value of a Complex Number

$|a + b\boldsymbol{i}| = \sqrt{a^2 + b^2}$

Polar Form of a Complex Number

$r(\cos\theta + \boldsymbol{i}\sin\theta)$

Product of Complex Numbers in Polar Form

$r_1(\cos\theta_1 + \boldsymbol{i}\sin\theta_1) \cdot r_2(\cos\theta_2 + \boldsymbol{i}\sin\theta_2) = r_1r_2[\cos(\theta_1 + \theta_2) + \boldsymbol{i}\sin(\theta_1 + \theta_2)]$

(continued)

FORMULAS

Quotient of Complex Numbers in Polar Form

$\frac{r_1(\cos \theta_1 + \boldsymbol{i} \sin \theta_1)}{r_2(\cos \theta_2 + \boldsymbol{i} \sin \theta_2)} =$

$\frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + \boldsymbol{i} \sin(\theta_1 - \theta_2)]$

De Moivre's Theorem

$[r(\cos \theta + \boldsymbol{i} \sin \theta)]^n =$
$r^n(\cos n\theta + \boldsymbol{i} \sin n\theta)$

CHAPTER 10 (pp. 614–693)

Distance Formula for Two Points

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of a Line Segment

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Rotation Equations

$x = x' \cos \theta + y' \sin \theta$

$y = -x' \sin \theta + y' \cos \theta$

Angle of Rotation

$\theta = \frac{\pi}{4}$, if $A = C$ or $\tan 2\theta = \frac{B}{A - C}$, if $A \neq C$

CHAPTER 11 (pp. 694–755)

Exponential Growth or Decay Formulas

$N = N_o(1 + r)^t \qquad N = N_oe^{kt}$

Compound Interest Formula

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

Continuously Compounded Interest Formula

$A = Pe^{rt}$

Change of Base Formula

$\log_a n = \frac{\log_b n}{\log_b a}$

Doubling Time $t = \frac{\ln 2}{k}$

CHAPTER 12 (pp. 758–835)

***n*th Term of an Arithmetic Sequence**

$a_n = a_1 + (n - 1)d$

Sum of an Arithmetic Sequence

$S_n = \frac{n}{2}(a_1 + a_n)$

***n*th Term of a Geometric Sequence**

$a_n = a_1 r^{n-1}$

Sum of a Finite Geometric Sequence

$S_n = \frac{a_1 - a_1r^n}{1 - r}$

Sum of an Infinite Geometric Series

$S_n = \frac{a_1}{1 - r}$

Euler's Formula

$e^{i\alpha} = \cos \alpha + \boldsymbol{i} \sin \alpha$

CHAPTER 13 (pp. 836–887)

Definitions of Permutations

$P(n, n) = n! \qquad P(n, r) = \frac{n!}{(n - r)!}$

Definition of Combination

$C(n, r) = \frac{n!}{(n - r)!\, r!}$

Permutation of *n* objects with repetitions (*p* alike and *q* alike)

$\frac{n!}{p!\, q!}$

Circular Permutations of *n* Objects

$\frac{n!}{n}$ or $(n - 1)!$

Probability of Two Independent Events

$P(A \text{ and } B) = P(A) \cdot P(B)$

Probability of Two Dependent Events

$P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Probability of Mutually Exclusive Events

$P(A \text{ or } B) = P(A) + P(B)$

Probability of Inclusive Events

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Conditional Probability

$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$ where $P(B) \neq 0$

CHAPTER 14 (pp. 888–939)

Arithmetic Mean $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

Mean of the Data in Frequency Distribution

$\overline{X} = \frac{\sum_{i=1}^{k}(f_i \cdot X_i)}{\sum_{i=1}^{k} f_i}$

Semi-Interquartile Range

$Q_R = \frac{Q_3 - Q_1}{2}$

Mean Deviation

$MD = \frac{1}{n}\sum_{i=1}^{n} |X_i - \overline{X}|$

Standard Deviation

$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2}$

Standard Deviation of the Data in a Frequency Distribution

$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2 \cdot f_i}{\sum_{i=1}^{n} f_i}}$

Standard Error of the Mean

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$

CHAPTER 15 (pp. 941–983)

Constant Multiple of a Power Rule for Derivatives

If $f(x) = cx^n$, where c is a constant and n is a rational number, then $f'(x) = cnx^{n-1}$.

Constant Multiple of a Power Rule for Antiderivatives

If $f(x) = kx^n$, where n is a rational number other than -1 and k is a constant, the antiderivative is

$F(x) = k \cdot \frac{1}{n + 1}x^{n+1} + C.$

Definite Integral

$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$ where

$\Delta x = \frac{b - a}{n}$

Fundamental Theorem of Calculus

$\int_a^b f(x)\,dx = F(b) - F(a)$, where

$F(x)$ is the antiderivative of $f(x)$.

SYMBOLS

$=$	is equal to
$\neq$	is not equal to
$<$	is less than
$\leq$	is less than or equal to
$\nless$	is not less than
$>$	is greater than
$\geq$	is greater than or equal to
$\ngtr$	is not greater than
$\approx$	is approximately equal to
$\{\ \}$	set notation
$\pm$	plus or minus
$\mp$	minus or plus
$\triangle ABC$	triangle ABC
RTS	arc RTS
$\angle ABC$	angle ABC
$m\angle ABC$	measure of angle ABC
AB	measure of line segment AB
$\overline{AB}$	line segment AB
$\lvert x \rvert$	the absolute value of x
x^n	the nth power of x
$\sqrt{x}$	the square root of x
$\sqrt[n]{x}$ or $x^{\frac{1}{n}}$	the nth root of x
$[\![x]\!]$	greatest integer not greater than x
A^{-1}	inverse of matrix A
a_{ij}	the element of the ith row and the jth column of matrix A
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$	the determinant $a_1b_2 - a_2b_1$
$\sin^{-1} x$	arcsin x
$\sin^2 \theta$	$(\sin \theta)^2$
∞	infinity
$\boldsymbol{i}$	$\sqrt{-1}$
$r \text{ cis } \theta$	$r(\cos \theta + \boldsymbol{i} \sin \theta)$
$\vec{\mathbf{v}}$ or $\overrightarrow{AB}$	a vector or directed line segment
$\langle x, y \rangle$	vector with initial point at origin and terminal point at (x, y)
$\lvert \vec{\mathbf{v}} \rvert$	magnitude of the vector $\vec{\mathbf{v}}$
$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$	inner product of vectors **a** and **b**
$\vec{\mathbf{a}} \times \vec{\mathbf{b}}$	cross product of vectors **a** and **b**
e	base of natural logarithms; ≈ 2.718
$n!$	n factorial
$\ln x$	logarithm of x with base e; natural logarithm
$\log_a x$	logarithm of x with base a
$\log x$	logarithm of x with base 10; common logarithm
$\lim\limits_{x \to a}$	the limit as x approaches a
$P(n, r)$	number of permutations of n objects, taken r at a time
$C(n, r)$	number of combinations of n objects, taken r at a time
$\overline{X}$	X bar or arithmetic mean
M_d	median
$\sigma_{\overline{X}}$	standard error of the mean
MD	mean deviation
$f(x)$	f of x or the value of function f at x
$f'(x)$	f prime of x or the derivative of $f(x)$
$\frac{dy}{dx}$	derivative of y
$f \circ g$ or $f(g(x))$	composite of functions f and g
$\int f(x)\, dx$	indefinite integral
$\int_a^b f(x)\, dx$	definite integral
α	alpha
β	beta
γ	gamma
Δ or δ	delta
ε	epsilon
θ	theta
λ	lambda
μ	mu
π	pi
σ	sigma; standard deviation
$\sum$	sigma; summation symbol
ϕ	phi
ω	omega; angular velocity